Hawking radiation of Black Holes

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Content

1 Introduction

In this paper I review the Hawking radiation of Black Holes.

According to Einstein's general relativity, a black hole exists and it absorbs everything. But, this property is just its classical aspect. In quantum theory a black hole would also emit. Hawking considered a quantum field in the classical spacetime background of a dust cloud that collapses to form a black hole[1]. He found that if no particles of the quantum field are present at early times, a distant observer at late times will detect an outgoing flux of particles having thermal spectrum. The flux of energy emitted by the black hole implies that its mass will decrease. Thus black holes are said to evaporate.

I describe the contents of my Thesis below.
In Section 2, I reviewed the Schwarzschild black hole and its geometrical properties. In Section 3, I explained Quantum Field Theory in Curved Spacetime(QFTCS) which Hawking used to reach his conclusion. In this theory the quantum fields satisfy their equations of motion, though not with the Minkowski metric but with the classical metric of a curved spacetime. The gravitational field is not quantized in the QFTCS. Another difference between the QFT and QFTCS is asymptotic completeness. The assumption that the Fock spaces of in-states, intermediate states and out-states are equal, is not correct. Thus, it is necessary to introduce the Bogoliubov transformation, in order to interpret particle creation in the out-states. I described them in Section 4. In Section 5 I derived the Hawking formula and described the physical interpretation of the Hawking radiation in the Schwarzschild black hole. The Hawking formula is [1]

$$T = \frac{\hbar c^3}{8\pi k_{\rm B} G M}, \tag{1}$$

where $T, \hbar, c, k_{\rm B}, G, M$ are the temperature of a black hole, the Planck constant, the speed of light, the Boltzmann constant, the Newton constant, and the mass of the black hole. This formula shows that the Schwarzschild black hole is a gray body of that temperature. The $\hbar$ in (1) implies that the Hawking radiation is caused by the quantum effect. Taking a classical limit, $\hbar \to 0$ yields $T \to 0$. This means that the classical temperature of a black hole is zero, and, therefore, there is no radiation classically, which is consistent with general relativity. It is important to confirm the Hawking radiation observationally. If the mass of a black hole is 10^{15} [g]($\leq$ stellarmass), the lifetime, the time for a black hole to disappear, is about the age of the Universe. So it is very hard to find an evaporating black hole in the Universe. It is suggested that the some black holes may have a short lifetime. They are called "mini" black holes. I briefly explained the lifetime of black holes and mini black holes in Section 6. Section 7 is the conclusion, and Section 8 is the outline where I shortly introduced the unsolved problems, especially, the black hole information paradox and Bekenstein-Hawking entropy.

In this Thesis I choose the natural units $G = 1, c = 1, \hbar = 1$.

2 Black Holes

In this section I describe "classical" aspects of black holes. The term "classical" means that a particle which travels in spacetime is not quantized and gravity is also not quantized.

Black holes are astronomical objects, which absorb everything. A black hole is an important object because it is as a prediction of general relativity. If it becomes clear that a black hole exists, that observation would make general relativity even more exciting.

Let us consider an isolated body which is widely dispersed and has sufficiently low density at early times collapses, and the entire spacetime is nearly flat. When the radius of this body becomes less than its Schwarzschild radius $R_{\rm SR}$ defined by

$$R_{\rm SR} = \frac{2GM}{c^2}, \tag{2}$$

the isolated body is called a black hole. Here c, G, M are the speed of light, the Newton constant and the mass of the body. The Schwarzschild radius of the sun is $R_{\rm SR\ sun} \approx 3$ [km]. The radius of the sun is $R_\odot \approx 7.0 \times 10^5$ [km], so $R_{\rm SR\ sun}/R_\odot \sim 4 \times 10^{-6}$. How small the Schwarzschild radius is! This lets us wonder that a black hole is a rare astronomical object and, maybe, doesn't exist. However, some indirect evidence has been submitted[12]. For instance, it is believed that the center of the Virgo galaxies M84 and M87(see Fig.1), about 50 million light years from the Earth, is a black hole. By looking at the central region of the galaxies with a spectrograph onboard the Hubble Space Telescope, a rapid rotation of gas, $v_r \sim 550$ [km/s], has been observed 60 light years from the core. Observational evidence for stellar mass black holes is sought for by searching for collapsed stars in binary systems. It would appear as the visible star orbits a compact, invisible, object with a mass larger than the maximum mass for a neutron star. Candidates for such a system exist: for example, the X-ray source Cygnus X-1(see Fig.2). According to the No-hair theorem[4], a black hole at late time is only characterized by its mass, angular momentum, and charge. When the angular momentum and charge are zero, we have a spherically symmetric black hole, which is called a Schwarzschild black hole. In the following discussion, I consider the geometries of Schwarzschild black holes only, for simplicity.
This section is based on [2], [5].

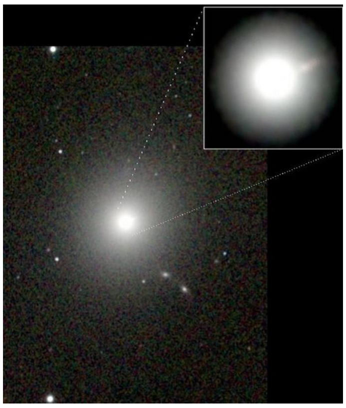

Fig. 1: M87 by 2MASS [13]

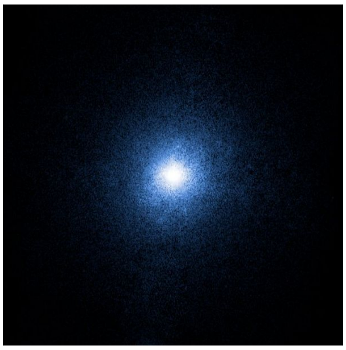

Fig. 2: Cygnus X-1 by CHANDRA [14]

2.1 Schwarzschild black hole

From [2] and [5], the Schwarzshild metric is

$$\mathrm{d}s^2 = \left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 - \left(1 - \frac{2M}{r}\right)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2, \tag{3}$$

with $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2$.

On radial null geodesics in Schwarzschild metric

$$\mathrm{d}t^2 = \frac{1}{\left(1 - \frac{2M}{r}\right)^2}\mathrm{d}r^2 =: \mathrm{d}R^2, \tag{4}$$

where

$$R = r + 2M\ln\left|\frac{r - 2M}{M}\right| \tag{5}$$

is the Regge-Wheeler radial coordinate. As r ranges from $2M$ to $+\infty$, R ranges from $-\infty$ to $+\infty$(Fig.3). For $r > 2M$, this mapping is monotonic.

Using this coordinate, (4) becomes

$$\mathrm{d}(t \pm R) = 0 \tag{6}$$

on radial geodesics. Defining the *incoming* radial null coordinate v by

$$v = t + R, \quad -\infty < v < +\infty \tag{7}$$

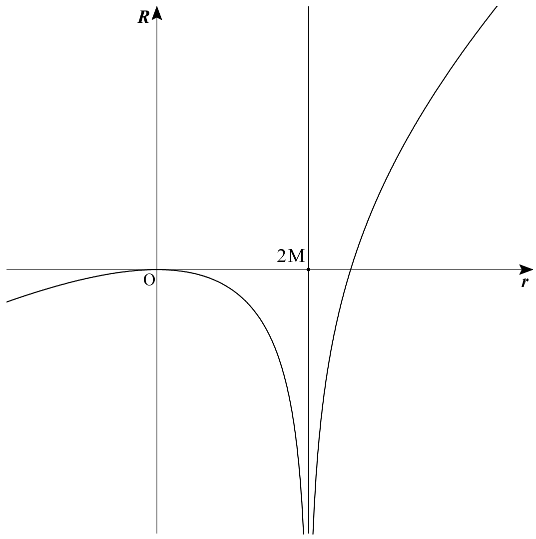

Fig. 3: the relation of R and r

and rewriting the Schwarzschild metric in the *incoming* Eddington-Finkelstein(EF) coordinates (v, r, θ, ϕ), we get

$$\begin{aligned} \mathrm{d}s^2 &= \left(1 - \frac{2M}{r}\right)(\mathrm{d}t^2 - \mathrm{d}R^2) - r^2\mathrm{d}\Omega^2 & (8) \\ &= \left(1 - \frac{2M}{r}\right)\mathrm{d}v^2 - 2\mathrm{d}r\mathrm{d}v - r^2\mathrm{d}\Omega^2. & (9) \end{aligned}$$

The Schwarzschild metric (3) has a singular point at $r = 2M$, but the metric (9) has no singularity. This means that the point $r = 2M$ is not a *real* singular point in Schwarzschild spacetime, it is just a coordinate singularity.

How about the *outgoing* EF coordinates? Defining the *outgoing* radial null coordinate u by

$$u = t - R, \quad -\infty < u < +\infty \tag{10}$$

and rewriting the Schwarzschild metric in *outgoing* Eddington-Finkelstein(EF) coordinates (u, r, θ, ϕ), we have

$$\mathrm{d}s^2 = \left(1 - \frac{2M}{r}\right)\mathrm{d}u^2 + 2\mathrm{d}r\mathrm{d}u - r^2\mathrm{d}\Omega^2. \tag{11}$$

The Schwarzschild metric in the *ingoing* and *outgoing* EF coordinates is initially defined only for $r > 2M$, but it can be analytically continued to all $r > 0$, because there is no singular point in this spacetime for $r > 0$. However, the $r < 2M$ region in the *outgoing* EF coordinates is *not* the same as the $r < 2M$ region in the *incoming* EF coordinates. To see this, note that for $r \leq 2M$ and the *incoming* EF coordinates, we have

$$\begin{aligned} 2\mathrm{d}r\mathrm{d}v &= -\left[\mathrm{d}s^2 + \left(\frac{2M}{r} - 1\right)\mathrm{d}v^2 + r^2\mathrm{d}\Omega^2\right] \\ &\leq 0, \quad \text{when } \mathrm{d}s^2 \geq 0. \end{aligned} \tag{12}$$

For the *outgoing* EF coordinates,

$$\begin{aligned} 2\mathrm{d}r\mathrm{d}u &= \mathrm{d}s^2 + \left(\frac{2M}{r} - 1\right)\mathrm{d}u^2 + r^2\mathrm{d}\Omega^2 \\ &\geq 0, \quad \text{when } \mathrm{d}s^2 \geq 0. \end{aligned} \tag{13}$$

Therefore, $\mathrm{d}r\mathrm{d}v \leq 0$ on timelike or null worldlines for the *incoming* EF coordinates, and $\mathrm{d}v > 0$ for future-directed worldlines, so $\mathrm{d}r \leq 0$. This shows that a star with a radius shorter than $2M$ must collapse. This is consistent with forming a black hole. In contrast, for the *outgoing* EF coordinates, $\mathrm{d}r \geq 0$. This means that a star with a radius shorter than $2M$ must *expand*. This is a white hole, the time reverse of a black hole. Both black and white holes are allowed by general relativity because of the time reversibility of Einstein's equations. But white holes require very special initial conditions near the sigularity, so that only black holes can occur in Nature(cf. irreversibility in thermodynamics).

2.1.1 Kruskal-Szekeres Coordinates

The $r > 2M$ region is covered by both the *incoming* and *outgoing* EF coodinates. We write the Schwarzschild metric in terms of (u, v, θ, ϕ) as

$$\mathrm{d}s^2 = \left(1 - \frac{2M}{r}\right)\mathrm{d}u\mathrm{d}v - r^2\mathrm{d}\Omega^2. \tag{14}$$

We now introduce the new coordinates (U, V) defined (for $r > 2M$) by

$$U = -\mathrm{e}^{-u/4M}, \quad V = \mathrm{e}^{v/4M}, \tag{15}$$

in terms of which the metric is

$$\mathrm{d}s^2 = \frac{32M^3}{r}\mathrm{e}^{-r/2M}\mathrm{d}U\mathrm{d}V - r^2\mathrm{d}\Omega^2. \tag{16}$$

The $r(U,V)$ is given implicitly by

$$UV = -\mathrm{e}^{R/2M} \Longleftrightarrow UV = -\left(\frac{r-2M}{2M}\right)\mathrm{e}^{r/2M}. \tag{17}$$

These are the Kruskal-Szekeres(KS) coordinates (U, V, θ, ϕ). Initially the metric is defined for $U < 0$ and $V > 0$, but it can be analytically continued to the $U > 0$ and $V < 0$ region. Now $r = 2M$ corresponds to $UV = 0$, and the singularity at $r = 0$ corresponds to $UV = 1$.

It is convenient to plot the lines of U = const. and V = const.(the *outgoing* and *incoming* radial null geodesics) at $\pi/4$ radian. The spacetime diagram looks like Fig.4. There are four regions of KS spacetime, depending

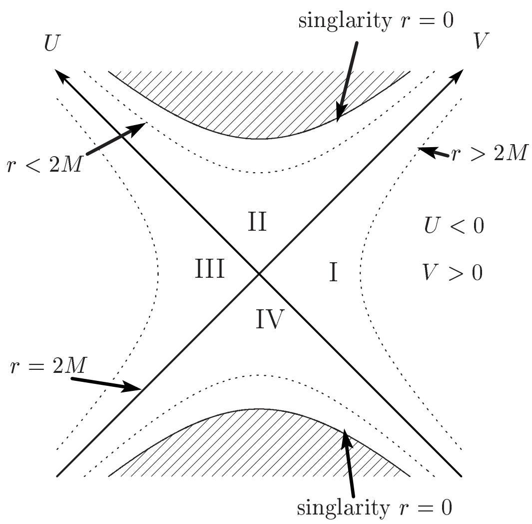

Fig. 4: the spacetime diagram in KS coordinates

on the signs of U and V. The regions I and II are covered by the *incoming* EF coordinates. These are the only regions relevant to gravitational collapse because the regions III and IV represent a white hole.

Let's derive the radial null geodesic $x^\mu(\lambda)$. The geodesics $x^\mu(\lambda)$ can be affinely parameterized. In general, geodesics are defined by

$$\frac{\mathrm{D}}{\mathrm{d}\lambda}\left(\frac{\mathrm{d}x^\mu}{\mathrm{d}\lambda}\right) = 0, \tag{18}$$

where $\mathrm{D}/\mathrm{d}\lambda$ is the covariant derivative along the geodesic with respect to λ. The action S_p of a particle is

$$S_p = \int_a^b \mathcal{L}\mathrm{d}\lambda, \tag{19}$$

where

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu} \frac{\mathrm{d}x^\mu}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\nu}{\mathrm{d}\lambda}. \tag{20}$$

If $g_{\mu\nu}$ and, hence, $\mathcal{L}$ are independent of some x^μ, then the conjugated momentum

$$p_\mu := \frac{\partial \mathcal{L}}{\partial(\mathrm{d}x^\mu/\mathrm{d}\lambda)} = g_{\mu\nu}\frac{\mathrm{d}x^\nu}{\mathrm{d}\lambda}, \tag{21}$$

is constant along the geodesics.

One finds that in the Schwarzschild metric, with the plane containing the geodesic taken at $\theta = \pi/2$, the constant momenta p_t and p_ϕ are

$$\left(1 - \frac{2M}{r}\right)\frac{\mathrm{d}t}{\mathrm{d}\lambda} = E, \tag{22}$$

$$r^2\frac{\mathrm{d}\phi}{\mathrm{d}\lambda} = L. \tag{23}$$

This expression is in terms fo (t, r, θ, ϕ) coordinates, E and L are energy and angular momentum, respectively. Because of on-shell condition, we have

$$\left(\frac{\mathrm{d}r}{\mathrm{d}\lambda}\right)^2 + \frac{L^2}{r^2}\left(1 - \frac{2M}{r}\right) = E^2. \tag{24}$$

The radial null geodesics are those with $L = 0$. They are characterized by (22) and

$$\frac{\mathrm{d}r}{\mathrm{d}\lambda} = \pm E, \tag{25}$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}\lambda} = 0. \tag{26}$$

From these equations, we get

$$\frac{\mathrm{d}t}{\mathrm{d}\lambda} \mp \left(1 - \frac{2M}{r}\right)^{-1} \frac{\mathrm{d}r}{\mathrm{d}\lambda} = 0 \tag{27}$$

or

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}(t \mp R) = 0, \tag{28}$$

where R is defined by

$$\frac{\mathrm{d}R}{\mathrm{d}r} = \left(1 - \frac{2M}{r}\right)^{-1}. \tag{29}$$

As $r \to 2M$ from above, $R \to -\infty$. Along any out going radial null geodesic the null coordinate

$$u := t - R \tag{30}$$

is constant. Along any incoming radial null geodesic the null coordinate

$$v := t + R \tag{31}$$

is constant. These are the *outgoing* and *incoming* EF coordinates again.

Let $\mathcal{C}$ be an incoming radial null geodesic defined by $v = v_1$ for some v_1, that passes through the event horizon of the Schwarzshild black hole. Along the null geodesic $\mathcal{C}$,

$$\frac{\mathrm{d}u}{\mathrm{d}\lambda} = \frac{\mathrm{d}t}{\mathrm{d}\lambda} - \frac{\mathrm{d}R}{\mathrm{d}\lambda} = 2\left(1 - \frac{2M}{r}\right)^{-1} E. \tag{32}$$

From (25), $\mathrm{d}r/\mathrm{d}\lambda = -E$ along C, so

$$r - 2M = -E\lambda, \tag{33}$$

where λ is taken to be zero at the event horizon $r = 2M$. For $r > 2M$, the affine parameter λ is negative. It follows that

$$\left(1 - \frac{2M}{r}\right)^{-1} = 1 - \frac{2M}{E\lambda}, \tag{34}$$

$$\frac{\mathrm{d}u}{\mathrm{d}\lambda} = 2E - \frac{4M}{\lambda}. \tag{35}$$

Therefore, we get an expression of the *incoming* null geodesic C,

$$u = 2E\lambda - 4M \ln \frac{\lambda}{K_1}, \tag{36}$$

where K_1 is a negative constant. Far away from the event horizon,

$$u \approx 2E\lambda, \tag{37}$$

while near the event horizon($\lambda = 0$),

$$u \approx -4M \ln \frac{\lambda}{K_1}. \tag{38}$$

So the *incoming* EF coordinate $u \to -\infty$ at the past null infinity and $u \to +\infty$ at the event horizon. In the EF coordinates (u, v, θ, ϕ), when $\lambda >> 1$, v becomes an affine parameter because of (37). Therefore, $v - v_0$ must be related to the separation λ between $u(v)$ and $u(v_0)$ at the past null infinity by

$$v_0 - v = K_2\lambda, \tag{39}$$

where K_2 is a negative constant. Hence, near the event horizon,

$$u \approx -4M \ln \left(\frac{v_0 - v}{K_1 K_2} \right). \tag{40}$$

This geodesic curve contributes to particle creation at most. Here the product $K_1 K_2$ is a positive constant parameter.

2.1.2 Carter-Penrose diagram in the Schwarzschild metric

It is convenient to use a Carter-Penrose(CP) diagram[5], [7], [9]. In the Schwarzschild metric, the CP diagram is like Fig.5, where radial null geodesics are represented by lines at $\pm 45°$ angles. A conformal transformation was made, so infinity can be represented on a CP diagram also.

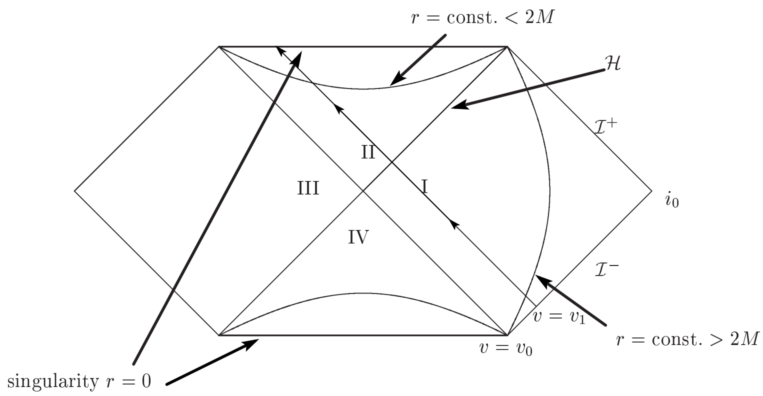

Fig. 5: Carter-Penrose diagram in the Schwarzshild spacetime

$\mathcal{I}^-$ in the Figure represents the past null infinity, from which *incoming* null geodesics originate. It is labeled by the arrow showing the direction in which the null coordinate v increases along $\mathcal{I}^-$. the future null infinity, $\mathcal{I}^+$, where *outgoing* null geodesics terminate, is labeled by the arrow showing the direction in which the null coordinate u increases along $\mathcal{I}^+$. The spatial infinity is, i_0, where t = finite and $r \to \infty$. The ray $\mathcal{C}$ is an *incoming* null geodesic when $v = v_1$, that enters the black hole through the event horizon, $\mathcal{H}$, which is the labeled null ray running from $r = 0$ to the point at which the singularity and $\mathcal{I}^+$ intersect in Fig.5. The *incoming* ray with $v = v_0$ is the last one that passes through the center of the body and reaches $\mathcal{I}^+$. The *incoming* null rays with $v > v_0$ enter the black hole through $\mathcal{H}$, and run into the singularity.

3 QFT in flat and curved spacetimes

In this section, I introduce QFTCS according to [7], [8], [9]. As was mentioned in Introduction, only matter fields are quantized in QFTCS, and the gravitational field is not quantized. Basically, the QFTCS is obtained by replacing the Minkowski metric $\eta_{\mu\nu}$ in QFT with a curved spacetime metric $g_{\mu\nu}$. Here I only use a free quantized scalar field for simplicity because it becomes essentially the same discussion for fermions and gauge fields.

3.1 Free Scalar QFT in curved spacetime

The invariant line element is

$$\mathrm{d}s^2 = g_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu,$$

where the signature is $(+,-,-,-)$ but the metric $g_{\mu\nu}$ is not flat. The action S is constructed from a scalar field ϕ as

$$S = \int \mathrm{d}^4x \mathcal{L} \tag{41}$$

with

$$\mathcal{L} = \frac{1}{2}|g|^{1/2}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2), \tag{42}$$

where $g = \det g_{\mu\nu}$, $g^{\mu\nu} = (g_{\mu\nu})^{-1}$ and m is a mass of the scalar field. The equation of motion is

$$(\Box + m^2)\phi = 0, \tag{43}$$

where $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$ and ∇_μ is a covariant derivative. Because of (43), ϕ can be expanded by solutions of (43),

$$\phi(x) = \int \mathrm{d}^3\mathbf{k}(A_\mathbf{k} f_\mathbf{k}(x) + A^\dagger_\mathbf{k} f^*_\mathbf{k}(x)), \tag{44}$$

where $f_\mathbf{k}(x)$ is a solution to (43). The inner product in the solution space of the Klein-Gordon equation is defined by

$$(\phi_1, \phi_2) = \mathrm{i}\int_\Sigma \mathrm{d}^3x|g|^{1/2}g^{0\nu}(\phi_1^*\partial_\nu\phi_2 - \phi_2^*\partial_\nu\phi_1), \tag{45}$$

where Σ is a $t=$ const. hypersurface. If ϕ_1 and ϕ_2 are solutions of EOM(43) which vanish at spatial infinity, (ϕ_1, ϕ_2) is conserved. So $f_{\mathbf{k}}$ and $f_{\mathbf{k}'}$ satisfy

$$\frac{\mathrm{d}}{\mathrm{d}t}(f_{\mathbf{k}}, f_{\mathbf{k}'}) = 0, \tag{46}$$

and also

$$(f_{\mathbf{k}}, f_{\mathbf{k}'}) = \delta(\mathbf{k}-\mathbf{k}'), \quad (f^*_{\mathbf{k}}, f^*_{\mathbf{k}'}) = -\delta(\mathbf{k}-\mathbf{k}'), \quad (f_{\mathbf{k}}, f^*_{\mathbf{k}'}) = 0. \tag{47}$$

The $(f_{\mathbf{k}})_{\mathbf{k}\in\mathbb{R}}$ is the orthonormal basis in the solution space. This basis is non-unique and after canonical quantization of fields, there will be different notions of vacuum, according to each different orthonomal basis. So QFTCS has many vacua. It is the difference between QFT in flat space and QFTCS. This is because there is no preferred time coordinate in curved spacetime.

By (44) and (47), we have

$$A_{\mathbf{k}} = (f_{\mathbf{k}}, \phi) = \mathrm{i}\int_{\Sigma} \mathrm{d}^3x |g|^{1/2} g^{0\nu}(f^*_{\mathbf{k}}\partial_\nu\phi - \phi^*\partial_\nu f_{\mathbf{k}}). \tag{48}$$

A canonical commutation relation(CCR) to be imposed to quantize the scalar field,

$$\left[A_{\mathbf{k}}, A^\dagger_{\mathbf{k}'}\right] = \delta(\mathbf{k}-\mathbf{k}'), \quad [A_{\mathbf{k}}, A_{\mathbf{k}'}] = 0. \tag{49}$$

From this and (43), ϕ and $\pi := \partial\mathcal{L}/\partial\dot{\phi}$ satisfy the CCR:

$$[\phi(\mathbf{x},t), \pi(\mathbf{x}',t)] = \mathrm{i}\delta(\mathbf{x}-\mathbf{x}'), \quad [\phi(\mathbf{x},t), \phi(\mathbf{x}',t)] = 0, \quad [\pi(\mathbf{x},t), \pi(\mathbf{x}',t)] = 0. \tag{50}$$

4 Bogoliubov transformation

When an operator $a_{\mathbf{k}}, \mathbf{k}\in\mathbb{R}^3$ satisfies $\left[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}'}\right] = \delta(\mathbf{k}-\mathbf{k}')$ and $[a_{\mathbf{k}}, a_{\mathbf{k}'}] = 0$, we can consider a new operator $b_{\mathbf{k}}$ defined as $b_{\mathbf{k}} := \alpha_{\mathbf{k}} a_{\mathbf{k}} + \beta^*_{\mathbf{k}} a^\dagger_{-\mathbf{k}}$. If $\alpha_{\mathbf{k}}$ and $\beta_{\mathbf{k}}$ satisfy $|\alpha_{\mathbf{k}}|^2 - |\beta_{\mathbf{k}}|^2 = 1$, the transformation $a_{\mathbf{k}} \mapsto b_{\mathbf{k}}$ is called the Bogoliubov transformation[11].

Because of $|\alpha_{\mathbf{k}}|^2 - |\beta_{\mathbf{k}}|^2 = 1$, the commutation relation of $b_{\mathbf{k}}$ becomes

$$\left[b_{\mathbf{k}}, b^\dagger_{\mathbf{k}'}\right] = (|\alpha_{\mathbf{k}}|^2 - |\beta_{\mathbf{k}}|^2)\delta(\mathbf{k}-\mathbf{k}') = \delta(\mathbf{k}-\mathbf{k}').$$

So the Bogoliubov transformation preserves the CCR. This transformation is used to diagonalize Hamiltonian. When a physical system has finite number of degrees of freedom, the Bogoliubov transformation is unitary. However, if the number of dof is infinite, the transformation is not unitary. This means that the Hamiltonian written by $a_{\mathbf{k}}$ is a different system from that with the Hamiltonian in terms of $b_{\mathbf{k}}$.

5 Hawking radiation

In this section, I describe the Hawking radiation. First, I introduce the sandwich spacetime where particle creation occurs. It is the simplest example of the Hawking radiation. Next, I describe the Hawking radiation of the Schwarzschild black hole.
This section is based on [6], [7], [8].

5.1 Particle creation in the Sandwich spacetime

The sandwich spacetime metric is spatially flat and isotropically changing,

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a(t)^2 \left(\sum_i (\mathrm{d}x^i)^2 \right). \tag{51}$$

This metric is asymptotically static,

$$a(t) \sim \begin{cases} a_1 & (t \to -\infty) \\ a_2 & (t \to +\infty). \end{cases} \tag{52}$$

With this metric, EOM of a scalar field becomes

$$a^{-3}\partial_t(a^3 \partial_t \phi) - a^{-2} \sum_i \partial_i^2 \phi = 0, \tag{53}$$

where I took $m = 0$ for simplicity. Imposing the periodic boundary conditions, $\phi(x^i + L, t) = \phi(x^i, t)$, we can expand ϕ in the form

$$\phi(x) = \sum_{\mathbf{k}} (A_{\mathbf{k}} f_{\mathbf{k}}(x) + A_{\mathbf{k}}^\dagger f_{\mathbf{k}}^*(x)). \tag{54}$$

Here

$$f_{\mathbf{k}}(x) = V^{-1/2} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} \psi_k(\tau), \tag{55}$$

$V = L^3, k^i = 2\pi n^i / L, n^i \in \mathbb{Z}, k = |\mathbf{k}|$ and

$$\tau = \int^t a(t')^{-3} \mathrm{d}t'. \tag{56}$$

It follows from (53) that

$$\frac{\mathrm{d}^2\psi_k}{\mathrm{d}\tau^2} + k^2 a^4 \psi_k = 0. \tag{57}$$

By imposing the initial condition given by (52),when $t \to -\infty$,

$$f_{\mathbf{k}}(x) \sim (V {a_1}^3)^{-1/2}(2\omega_{1k})^{-1/2}\mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{x}-\omega_{1k}t)} \tag{58}$$

with $\omega_{1k} = k/a_1$. From this and (55),

$$\psi_k(\tau) \sim (2{a_1}^3\omega_{1k})^{-1/2}\mathrm{e}^{-\mathrm{i}\omega_{1k}{a_1}^3\tau} \tag{59}$$

as $t \to -\infty$.

Now I impose a canonical commutation relation(CCR) to quantize this scalar field,

$$\left[A_{\mathbf{k}}, A^\dagger_{\mathbf{k}'}\right] = \delta(\mathbf{k}-\mathbf{k}'), \quad [A_{\mathbf{k}}, A_{\mathbf{k}'}] = 0. \tag{60}$$

From this and (53), ϕ and $\pi := \partial\mathcal{L}/\partial\dot{\phi} = a3\partial_t\phi = \partial_\tau\phi$ satisfy CCR,

$$[\phi(\mathbf{x},t), \pi(\mathbf{x}',t)] = \mathrm{i}\delta(\mathbf{x}-\mathbf{x}'), \quad [\phi(\mathbf{x},t), \phi(\mathbf{x}',t)] = 0, \quad [\pi(\mathbf{x},t), \pi(\mathbf{x}',t)] = 0. \tag{61}$$

Now let's construct a Fock space of states. There is a Fock space on each $t =$ const. hyperplane, so we must distinguish all vaccua on hyperplanes. I call the vaccum on the initial condition $|0\rangle$ as

$$A_{\mathbf{k}}\,|0\rangle = 0, \quad {}^\forall\mathbf{k}. \tag{62}$$

The time development of ψ_k is governed by the ordinary second-order differential equation (57). It has two linearly independent solutions $\psi_k^{(\pm)}$ such that as $t \to +\infty$

$$\psi_k^{(\pm)}(\tau) \sim (2{a_2}^3\omega_{2k})^{-1/2}\mathrm{e}^{\mp\mathrm{i}\omega_{2k}{a_2}^3\tau}, \tag{63}$$

where $\omega_{2k} := k/a_2$. Therefore, ψ_k can be represented by

$$\psi_k(\tau) = \alpha_k\psi_k^{(+)}(\tau) + \beta_k\psi_k^{(-)}(\tau) \tag{64}$$

$$\psi_k(\tau) \sim (2{a_2}^3\omega_{2k})^{-1/2}\left[\alpha_k\mathrm{e}^{-\mathrm{i}\omega_{2k}{a_2}^3\tau} + \beta_k\mathrm{e}^{\mathrm{i}\omega_{2k}{a_2}^3\tau}\right], \quad (t \to +\infty). \tag{65}$$

The Wronskian of (57) gives the conserved quantity

$$\psi_k \partial_\tau \psi_k^* - \psi_k^* \partial_\tau \psi_k = \mathrm{i}, \tag{66}$$

It requires that

$$|\alpha_k|^2 - |\beta_k|^2 = 1. \tag{67}$$

From (55),(65), it follows that at late times

$$f_{\mathbf{k}}(x) \sim (V a_2{}^3)^{-1/2} (2\omega_{2k})^{-1/2} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} \left[\alpha_k \mathrm{e}^{-\mathrm{i}\omega_{2k}t} + \beta_k \mathrm{e}^{\mathrm{i}\omega_{2k}t}\right]. \tag{68}$$

On the other hand, ϕ can also be written as

$$\phi(x) = \sum_{\mathbf{k}} (a_{\mathbf{k}} g_{\mathbf{k}}(x) + a_{\mathbf{k}}^\dagger g_{\mathbf{k}}^*(x)) \tag{69}$$

with $g_{\mathbf{k}}$ being a solution of the field equation with a positive frequency at late times,

$$g_{\mathbf{k}}(x) \sim (V a_2{}^3)^{-1/2} (2\omega_{2k})^{-1/2} \mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{x} - \omega_{2k}t)} \tag{70}$$

and

$$a_{\mathbf{k}} := \alpha_{\mathbf{k}} A_{\mathbf{k}} + \beta_{\mathbf{k}}^* A_{-\mathbf{k}}^\dagger. \tag{71}$$

This yields

$$\left[a_{\mathbf{k}}, a_{\mathbf{k}'}^\dagger\right] = (|\alpha_k|^2 - |\beta_k|^2)\delta(\mathbf{k} - \mathbf{k}') = \delta(\mathbf{k} - \mathbf{k}'). \tag{72}$$

Therefore, $A_{\mathbf{k}} \mapsto a_{\mathbf{k}}$ is a Bogoliubov transformation.

Using the $a_{\mathbf{k}}$ and $|0\rangle$ we can calculate the expectation value of number of particles present at late times in mode $\mathbf{k}$:

$$\langle N_{\mathbf{k}} \rangle_{t \to +\infty} = \langle 0 | \, a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \, | 0 \rangle = |\beta_k|^2. \tag{73}$$

On the other hand, at early times

$$\langle N_{\mathbf{k}} \rangle_{t \to -\infty} = \langle 0 | \, A_{\mathbf{k}}^\dagger A_{\mathbf{k}} \, | 0 \rangle = 0. \tag{74}$$

Thus, if $a(t)$ is such that $|\beta_k|^2$ is non-zero, as is generally the case, particles are created by the changing scale factor of the universe! This is the simplest example of the Hawking radiation. An exact solution for a specific $a(t)$ is discussed in Section 2.8 of [7].

5.2 Particle creation in the Schwarzschild spacetime

A general QFTCS has many vacua, but we need only two vacua to calculate: the vacuum at the early time($\mathcal{I}^-$ in Fig. 5) and the vacuum at the late time($\mathcal{I}^+$ in Fig. 5). It is the same as a calculation of the S matrix in QFT in flat space.

We work in the Heisenberg picture. Let the state vector $|0\rangle$ be chosen to have no particles of the field *incoming* from $\mathcal{I}^-$. Thus, $|0\rangle$ is annihilated by the A_ω corresponding to particles *incoming* from $\mathcal{I}^-$:

$$A_\omega |0\rangle = 0. \tag{75}$$

As in the sandwich spacetime, the spectrum of *outgoing* particles is determined by the coefficients of the Bogoliubov transformation relating a_ω to A_ω and $A_\omega^\dagger$. I assume f_ω and g_ω are a complete set at $\mathcal{I}^-$ and at $\mathcal{I}^+$, respectively. We have

$$g_\omega = \int \mathrm{d}\omega' (\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f_{\omega'}^*), \tag{76}$$

where the $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$ are complex numbers, independent of the coordinates. From (47), they can be expressed as

$$\alpha_{\omega\omega'} = (f_{\omega'}, g_\omega), \tag{77}$$

$$\beta_{\omega\omega'} = -(f_{\omega'}^*, g_\omega). \tag{78}$$

Furthermore, using (47) and (76) it follows that

$$(g_{\omega_1}, g_{\omega_2}) = \int \mathrm{d}\omega' (\alpha_{\omega_1\omega'}^* \alpha_{\omega_2\omega'} - \beta_{\omega_1\omega'}^* \beta_{\omega_2\omega'}). \tag{79}$$

In the Schwarzschild metric, f_ω is

$$f_\omega \sim \omega^{-1/2} r^{-1} \mathrm{e}^{-\mathrm{i}\omega v} S(\theta, \phi), \tag{80}$$

where $S(\theta, \phi)$ is the spherical harmonics and discrete quantum numbers (l, n) have been suppressed, and $v = t + r$ is the *incoming* null coordinate at $\mathcal{I}^-$. The factor of $\omega^{-1/2}$ is required by the normalization of the scalar product. On the other hand, g_ω is

$$g_\omega \sim \omega^{-1/2} r^{-1} \mathrm{e}^{-\mathrm{i}\omega u(v)} S(\theta, \phi), \tag{81}$$

where again quantum numbers (l, m) have been suppressed, and $u = t - r$ is the *outgoing* null coordinate at $\mathcal{I}^+$. The value of v is related to the value of u by (40),

$$u(v) = -4M \ln\left(\frac{v_0 - v}{K_1 K_2}\right), \quad (v < v_0). \tag{82}$$

Because the geodesics passes through the center of the collapsing body just before the event horizon has formed, and emerges as an *incoming* geodesic characterized by a value of $v \nearrow v_0$.

Using these asymptotic forms, $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$ can be expressed as

$$\alpha_{\omega\omega'} = C \int_{-\infty}^{v_0} \mathrm{d}v \left(\frac{\omega'}{\omega}\right)^{1/2} \mathrm{e}^{\mathrm{i}\omega' v} \mathrm{e}^{-\mathrm{i}\omega u(v)}, \tag{83}$$

$$\beta_{\omega\omega'} = C \int_{-\infty}^{v_0} \mathrm{d}v \left(\frac{\omega'}{\omega}\right)^{1/2} \mathrm{e}^{-\mathrm{i}\omega' v} \mathrm{e}^{-\mathrm{i}\omega u(v)}. \tag{84}$$

Here C is a constant. Let us substitute (82), and let $s := v_0 - v$ in (83), $s := v - v_0$ in (84) and $K := K_1 K_2$. Then we have

$$\alpha_{\omega\omega'} = -C \int_{\infty}^{0} \mathrm{d}s \left(\frac{\omega'}{\omega}\right)^{1/2} \mathrm{e}^{-\mathrm{i}\omega' s} \mathrm{e}^{\mathrm{i}\omega' v_0} \exp\left[\mathrm{i}\omega 4M \ln\left(\frac{s}{K}\right)\right], \tag{85}$$

$$\beta_{\omega\omega'} = C \int_{-\infty}^{0} \mathrm{d}s \left(\frac{\omega'}{\omega}\right)^{1/2} \mathrm{e}^{-\mathrm{i}\omega' s} \mathrm{e}^{-\mathrm{i}\omega' v_0} \exp\left[\mathrm{i}\omega 4M \ln\left(-\frac{s}{K}\right)\right]. \tag{86}$$

In (85) the contour of integration along the real axis from 0 to ∞ in the complex s-plane can be joined by a quarter circle at infinity to the contour along the imaginary axis from $-\mathrm{i}\infty$ to 0. Because there are no poles of the integrand in the quadrant enclosed by the contour, and the integrand vanishes on the boundary at infinity, the integral from 0 to ∞ along the real s-axis equals to the integral from $-\mathrm{i}\infty$ to 0 along the imaginary s-axis. Thus, putting $s := \mathrm{i}s'$, (85) becomes

$$\alpha_{\omega\omega'} = -\mathrm{i}C \int_{-\infty}^{0} \mathrm{d}s' \left(\frac{\omega'}{\omega}\right)^{1/2} \mathrm{e}^{\omega' s'} \mathrm{e}^{\mathrm{i}\omega' v_0} \exp\left[\mathrm{i}\omega 4M \ln\left(\frac{\mathrm{i}s'}{K}\right)\right]. \tag{87}$$

Similarly, in (86), the integral along the real axis in the complex s-plane from $-\infty$ to 0 can be joined by a quarter circle at infinity to the contour along the imaginary s-axis from $-\mathrm{i}\infty$ to 0. As before, the integrals are equal. Therefore, putting $s := \mathrm{i}s'$, we have

$$\beta_{\omega\omega'} = \mathrm{i}C \int_{-\infty}^{0} \mathrm{d}s' \left(\frac{\omega'}{\omega}\right)^{1/2} \mathrm{e}^{\omega' s'} \mathrm{e}^{-\mathrm{i}\omega' v_0} \exp\left[\mathrm{i}\omega 4M \ln\left(\frac{-\mathrm{i}s'}{K}\right)\right]. \tag{88}$$

Taking the cut in the complex plane along the negative real axis to define a single-valued natural logarithm function, we find that for $s' < 0$ (as in these integrands),

$$\ln(\mathrm{i}s'/K) = \ln(-\mathrm{i}|s'|/K) = -\mathrm{i}(\pi/2) + \ln(|s'|/K), \tag{89}$$

$$\ln(-\mathrm{i}s'/K) = \ln(\mathrm{i}|s'|/K) = \mathrm{i}(\pi/2) + \ln(|s'|/K). \tag{90}$$

Then

$$\alpha_{\omega\omega'} = -\mathrm{i}C\mathrm{e}^{\mathrm{i}\omega' v_0}\mathrm{e}^{2\pi\omega M} \int_{-\infty}^{0} \mathrm{d}s' \left(\frac{\omega'}{\omega}\right)^{1/2} \mathrm{e}^{\omega' s'} \exp\left[\mathrm{i}\omega 4M \ln\left(\frac{|s'|}{K}\right)\right], \tag{91}$$

and

$$\beta_{\omega\omega'} = \mathrm{i}C\mathrm{e}^{-\mathrm{i}\omega' v_0}\mathrm{e}^{-2\pi\omega M} \int_{-\infty}^{0} \mathrm{d}s' \left(\frac{\omega'}{\omega}\right)^{1/2} \mathrm{e}^{\omega' s'} \exp\left[\mathrm{i}\omega 4M \ln\left(\frac{|s'|}{K}\right)\right]. \tag{92}$$

Hence, it follows that

$$|\alpha_{\omega\omega'}|^2 = \exp(8\pi M\omega)|\beta_{\omega\omega'}|^2. \tag{93}$$

For the components g_ω of this part of the wave packet, we have the scalar product

$$(g_{\omega_1}, g_{\omega_2}) = \Gamma(\omega_1)\delta(\omega_1 - \omega_2), \tag{94}$$

where $\Gamma(\omega_1)$ is the fraction of an out going packet of frequency ω_1 at $\mathcal{I}^+$ that would propagate backward in time through the collapsing body to $\mathcal{I}^-$. It follows from (79) and (94) that

$$\Gamma(\omega_1)\delta(\omega_1 - \omega_2) = \int \mathrm{d}\omega' (\alpha^*_{\omega_1\omega'}\alpha_{\omega_2\omega'} - \beta^*_{\omega_1\omega'}\beta_{\omega_2\omega'}). \tag{95}$$

The $a_\omega := (g_\omega, \phi)$ is of interest to us, because

$$\langle 0| a^\dagger_\omega a_\omega |0\rangle = \int \mathrm{d}\omega' |\beta_{\omega\omega'}|^2 \tag{96}$$

is the number of particles(with a frequency ω) created from the black hole. However, we encounter an infinity if we try to evaluate this quantity. The infinity is a consequence of the $\delta(\omega_1 - \omega_2)$ that appears in (94). We expect that $\langle 0| a^\dagger_\omega a_\omega |0\rangle$ is the total number of created particles per unit frequency that reach $\mathcal{I}^+$ at late times in the wave $g^{(2)}_\omega$. This total number is infinite because there is a steady flux of particles reaching $\mathcal{I}^+$. One heuristic way to see this is to replace $\delta(\omega_1 - \omega_2)$ in (95) by

$$\delta(\omega_1 - \omega_2) = \lim_{\tau\to\infty} \int_{\tau/2}^{\tau/2} \mathrm{d}t \exp[\mathrm{i}(\omega_1 - \omega_2)]. \tag{97}$$

Then, formally, we can write (95) when $\omega_1 = \omega_2 = \omega$, as

$$\begin{aligned} \lim_{\tau\to\infty} \Gamma(\omega)(\tau/2\pi) &= \int \mathrm{d}\omega'(|\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2) \\ &= [\exp(8\pi M\omega) - 1] \int \mathrm{d}\omega' |\beta_\omega|^2, \end{aligned} \tag{98}$$

where we have used (93). Hence, $\langle N_\omega\rangle_{t\to\infty} := \langle 0| a^\dagger_\omega a_\omega |0\rangle$ is given by

$$\langle N_\omega\rangle_{t\to\infty} = \lim_{\tau\to\infty} \frac{\tau}{2\pi} \frac{\Gamma(\omega)}{\mathrm{e}^{8\pi M\omega} - 1}. \tag{99}$$

The physical interpretation of this is that at late times, the number of created particles per unit angular frequency and per unit time that pass through a surface $r = R$(where R is much larger than the circumference of the black hole event horizon) is

$$\frac{1}{2\pi} \frac{\Gamma(\omega)}{\mathrm{e}^{8\pi M\omega} - 1}. \tag{100}$$

It implies that a Schwarzshild black hole emits and absorbs radiation exactly like a gray body of absorptivity $\Gamma(\omega)$ and temperature T given by

$$k_B T = (8\pi M)^{-1} = (2\pi)^{-1}\kappa, \tag{101}$$

where k_B is a Boltzmann's constant, and $\kappa = (4M)^{-1}$ is the surface gravity of the Schwarzshild black hole. Restoring $c, G, \hbar$, we have

$$T = \frac{\hbar c^3}{8\pi k_{\mathrm{B}} G M} \approx 6 \times 10^{-7} \frac{M_{\mathrm{sun}}}{M} \text{ [K]}. \tag{102}$$

Eq.(101) is the main result of Hawking[1]. So, why does a black hole emit particles? The answer is that the Hawking radiation is a quantum effect.

In other words, because vacuum bubbles of Feynman diagrams are no more bubbles(see Fig.6).

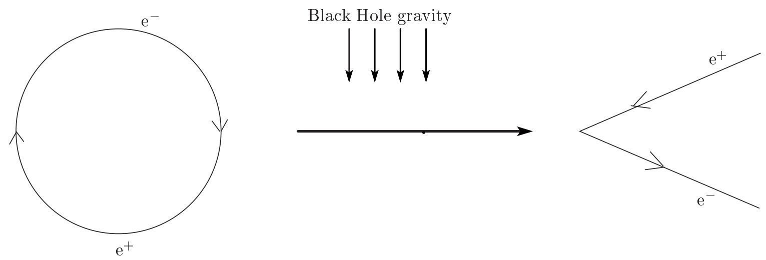

Fig. 6: A vacuum bubble diagram is breaking

In QFT in Minkowski spacetime, vacuum bubbles represent virtual particles that keep producing and annihilating. However, if there is a strong force, like a black hole gravity, vacuum bubbles break into a diagram which corresponds to a creation diagram. So pair-particles come from the vacuum! This diagram breaking occurs not only by black hole gravity but also by other strong force. For example, the Schwinger effect is one of the famous pair-particle production phenomena. This effect is caused by a strong electrical field $E \sim 10^{18}$V/m[15].

6 Phenomenology of Black Holes

In this Section I briefly describe phenomenology of black holes according to [12]. The Hawking formula (101) was derived in the previous Section. It is of great interest to study its consequences for physical and astronomical observations.

6.1 Black Hole lifetimes

In the previous Section, I derived the Hawking radiation. Once quantum effects are taken into account, black holes do radiate, therefore, black holes have finite lifetimes. According to quantum mechanics, pair production of virtual particles ($\Delta E = 2mc^2$)may take place in the vacuum, provided that they annihilate within the short time allowed by the uncertanity principle:$\Delta t \sim \hbar/2mc^2$. Such vacuum fluctuations will also take place in the neighbourhood of black holes. As was shown in the previous Section, one of pair-particles falls beyond the event horizon and the other becomes a "real" particle. In this process gravitational energy from the black hole is converted into the rest energy and the kinetic energy of the produced particle, so that energy of the order of ΔE is transferred from the black hole to the outside world. Thus, the time which a black hole evaporates, in other words, the lifetime τ of the black hole, is given by [12]

$$\tau \approx 10^{10} \left(\frac{M}{10^{15}[\mathrm{g}]} \right)^3 [\mathrm{years}]. \tag{103}$$

If $M \approx 10^{15}$[g], τ is about 10 billion years. This is nearly the same value as the age of the Universe, so that heavy black holes are expected to have a very long lifetime.

6.2 Mini Black Holes

It is conceivable that "mini" black holes(with $r_s \approx 10^{-15}$[m]) could have been formed during the Big Bang. Substituting this radius to the Schwarzschild radius formula, the upper limit mass of mini black holes is 10^{15}[g]. This is the reason I made a ratio in (103). The lower limit mass of mini black holes is also defined. It is the Planck mass $\sim 10^{-5}$[g]. Because the Planck mass is the energy scale in which the gravitational and quantum effect are samely important, and the QFTCS is the low energy effective theory of the quantum gravity. For the Planck mass, the lifetime is 10^{-43} [sec] $\sim$ the Planck time. This is the shortest possible time in physics, it is consistent with what I said above. From (103), black holes with masses much below 10^{15}[g] have a lifetime shorter than the present age of the Universe and should, therefore, have already been evaporated. Let us estimate the energy E of the gamma-rays radiated from a mini black hole. According to [12], it follows from (102) that

$$E \sim k_{\mathrm{B}}T \sim \hbar c^3/GM \sim 100 \left(\frac{10^{15}\ [\mathrm{g}]}{M} \right) [\mathrm{MeV}]. \tag{104}$$

Therefore, cosmic rays with energies around ~ 100 [MeV] are good candidates for the gamma-rays from mini black holes. There are some attempts to find mini black holes at the LHC or future colliders[16], [17]. It is not clear whether this interesting object, which includes elements of special and general relativity, quantum mechanics and thermodynamics, can be confirmed. Black

Holes and their Hawking evaporation are the theoretical laboratories where current ideas of quantum gravity are being tested.

7 Conclusion

I reviewed the Hawking radiation in this Thesis. First, I described the definition of a Schwarzschild black hole and its geometrical properties, and I showed some candidates for black holes. Second, I introduced quantum field theory in curved space is the main tool to analyze the Hawking radiation. Basically, QFTCS is obtained by replacing the flat metric with a curved metric. The existence of many vacua in QFTCS causes the particle creation in curved space. Third, I explained the Bogoliubov transformation needed to interpret a number of particles. Next, I described the Hawking radiation in the Sandwich spacetime and in the Schwarzschild black hole. For the Schwarzschild black hole, the number of created particles per unit angular frequency and per unit time that pass through a surface is given by

$$\frac{1}{2\pi}\frac{\Gamma(\omega)}{\mathrm{e}^{8\pi GM\omega/\hbar c^3}-1}.$$

Therefore, the Hawking temperature is

$$T=\frac{\hbar c^3}{8\pi k_{\mathrm{B}}GM}.$$

Finally, I briefly considered phenomenology of black holes including lifetimes of black holes and mini black holes. The lifetime of black holes is

$$\tau \approx 10^{10}\left(\frac{M}{10^{15}}\right)^3 \text{ [years]}.$$

The energy of the gamma-rays radiated from a mini black hole is

$$E \sim 100 \text{ [MeV]},$$

therefore, cosmic rays with energies around 100 [MeV] are good candidates for the gamma-rays from mini black holes. And there are attempts to find mini black holes at the LHC or in cosmic rays.

8 Outlook

In this section, I shortly introduce some fundamental unsolved problems, namely, the black hole information paradox and the Bekenstein-Hawking entropy. The following discussion is based on [4], [5] and [9].

8.1 Black Hole information paradox

When taking the Hawking radiation into account, a black hole will eventually evaporate, after which the spacetime has no event horizon. This is expressed by the following CP diagram:

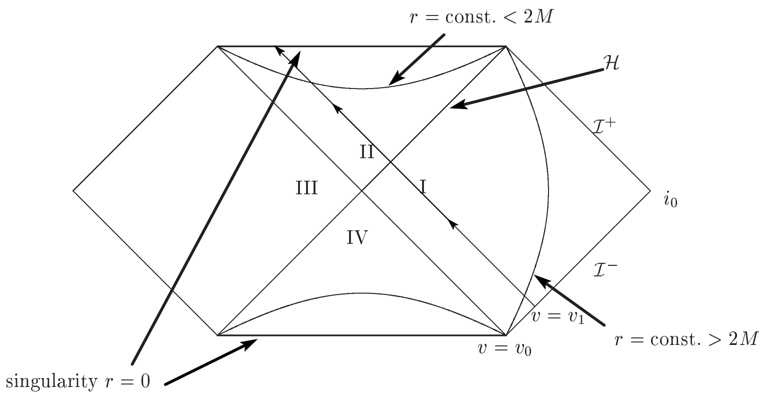

Fig. 7: Information loss paradox

Here $\mathcal{H}$ represents a new horizon. Σ_1 is a Cauchy surface[1] for this spacetime, but Σ_2 is not, because its past domain of dependence does not include the black hole region. Information from Σ_1 can propagate into the black hole region instead of Σ_2. Thus it appears that information is "lost" into the black hole. This implies a non-unitary evolution from Σ_1 to Σ_2, and, hence, the QFTCS is in conflict with the basic principle of quantum mechanics. To resolve this problem, the black hole complementarity is suggested in [18]. The idea is that the information is both reflected at the horizon $\mathcal{H}$ and passes through it without being able to escape, then no observer is able to confirm both pieces of information simultaneously. The authors of [18] assumed a

[1]Cauchy surface is the set of points which can be connected to every point in the spacetime by either future or past causal curves. See [2] for further details.

stretched horizon, a kind of membrane hovering outside the event horizon. The point is that, according to an infalling observer, nothing special happens at the horizon and in this case the information and the infalling observer hit the singularity. But, for an exterior observer, infalling information heats up the stretched horizon, which then re-radiates it as the Hawking radiation. This isn't to say that one has two copies of information (one copy goes inside, the other being Hawking radiated) — because of the the non-cloning theorem — this is not possible. What can be done is to detect information either inside the black hole or outside, but not both. In this sense, complementarity is to be taken in the quantum sense (like non-commuting observers).

Some ideas were suggested to solve this problem [21], [22].

8.2 Bekenstein-Hawking Entropy

The Hawking formula (102) means that we can define thermodynamics of black holes. Like the usual thermodynamics, the entropy of a black hole can be calculated [5]as

$$S = \frac{1}{4}A, \tag{105}$$

where A is the area of the event horizon. This formula is called the Bekenstein-Hawking entropy. In the usual thermodynamics, there is the microscopic interpretation of the entropy as

$$S = k_{\mathrm{B}} \ln W, \tag{106}$$

where W is the number of the states of a quantum system. What is the microscopic origin of the black hole entropy? For example, in [19], the Bekenstein-Hawking entropy was calculated by using the type II string theory on $K3 \times S^1$ for the so-called extremal(BPS like) black holes. However, realistic black holes are not extremal. The microscopic origin of the Bekenstein-Hawking entropy remains an outstanding problem, its microscopic interpretation in string theory requires a non-perturbative string thery which does not exist(yet). The perturbative string corrections to the Hawking temperature were computed in [20].

References

[1] S. Hawking, Particle Creation by Black Hole, Commun. Math. Phys. 43, 199-220(1975)

[2] S. Hawking and G.Ellis, The large scale structure of space-time, Cambridge University Press, Cambridge(1974)

[3] W. Pauli, theory of relativity, Dover Publications, New York(1981)

[4] J. Bekenstein, Black Holes: Classical Properties, Thermodynamics and Heuristic Quantization, arXiv:gr-qc/9808028v3

[5] P. Townsend, Black Holes, arXiv:gr-qc/9707012

[6] L. Parker, Quantized Fields and Particle creation in Expanding Universes. I, Phys. Rev., 183, 1057-1068(1969)

[7] L. Parker and D.Toms, Quantum Field Theory in Curved Spacetime, Cambridge University Press, Cambridge(2009)

[8] N. Birrell and P. Davis, Quantum field in curved space, Cambridge University Press, Cambridge(1984)

[9] P. Lambert, Introduction to Black Hole Evaporation, arXiv:gr-qc/1310.8312v1

[10] S. Weinberg, The Quantum Theory of Fields, Cambridge University Press, Cambridge(2005)

[11] Y. Takahashi, Quantum Field Theory I for researchers of physical property/Bussei kenkyusya no tame no ba no ryoushiron I(in Japanese), Baihuukan, Tokyo(1974)

[12] L.Bergström and A.Goobar, cosmology and particle astrophysics, Springer, Chichester(2006)

[13] http://www.ipac.caltech.edu/2mass/gallery/m87atlas.jpg, accessed Jan 14th

[14] http://chandra.harvard.edu/photo/2011/cygx1/, accessed Jan 14th

[15] G. Dunne, New Strong-Field QED Effects at ELI: Nonperturbative Vacuum Pair Production, arXiv:hep-th/0812.3163v2

[16] A. Barrau and J. Grain and S. Alexeyev, Gauss-Bonnet Black Holes at the LHC, arXiv:hep-th/0311238v2

[17] A. Barrau and C. Feron and J. Grain, Astrophysical production of microscopic black holes in a low Planck-scale World, arXiv:astro-ph/0505436v1

[18] L. Susskind and L. Thorlacius and J. Uglum, The stretched horizon and black hole complementarity, Phys. Rev D 48, 3743-3761 (1993)

[19] A. Strominger and C. Vafa, Microscopic Origin of the Beckenstein Hawking Entropy, arXiv:hep-th/9601029v2

[20] S. Ketov and S. Mochalov, Curvature cubed string correction to the Hawking temperature of a black hole, Phys. Lett. B 261, 35-40(1991)

[21] A. Almheiri, D. Marolf, J. Polchinski and J. Sully, Black Holes: Complementarity or Firewalls?, J. High Energy Phys. 2, 062 (2013)

[22] S. Hawking, Information Preservation and Weather Forecasting for Black Holes, arXiv:hep-th/1401.5761v1